ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

74. Proposed by NELSON S. RORAY, South Jersey Institute, Bridgeton, N. J.

Solve according to the conditions given :

$$\sqrt{x+1}+\sqrt{x}=\frac{3}{\sqrt{1+x}}$$

First, square without transposing and then solve; second, transpose $\sqrt{x+1}$ and then solve. Obtain the same roots as in the first way of solving.

I. Solution by J. M. BOORMAN, Counselor, Inventor, etc., etc., Hewlett, L. I., N. Y.

Solve ("conditions given") $\sqrt{x+1}+\sqrt{x}=\dfrac{3}{\sqrt{1+x}}$ . . . . (A).

The equation is of first degree. $\therefore$ *can have but one root, e. g.*

FIRST. The *conditioned* operation gives, $2(x+1)+2\sqrt{(x+1)}\sqrt{x}=1+\dfrac{9}{1+x}$. Thence $\sqrt{x+1}.\sqrt{x}=-(x+1)+\frac{1}{2}+\dfrac{9}{2(1+x)}$.

Square, etc., and reduce : $\therefore 8\frac{3}{4}(1+x)^2-4\frac{1}{2}(1+x)=20\frac{1}{4}$.

Divide and supply : $\therefore (1+x)^2-\frac{18}{35}(1+x)+\frac{81}{1225}=\frac{2916}{1225}$.

$\therefore x=\frac{4}{5}$; $x_1=-\frac{16}{7}$.

BUT, apply x_1 to the *given equation*. $\therefore \left(\dfrac{3+4}{\sqrt{7}}\right)\sqrt{-1}=\dfrac{3\sqrt{7}}{3\sqrt{-1}}=\dfrac{\sqrt{7}}{\sqrt{-1}}$.

Now $\dfrac{3+4}{\sqrt{7}}=\sqrt{7}$. $\therefore \sqrt{7}\sqrt{-1}=\sqrt{7}\left(\dfrac{1}{\sqrt{-1}}\right)$.

$\therefore -1\sqrt{7}=\sqrt{7}$; or $2\sqrt{7}=0$! ! So $x_1=-\frac{16}{7}$ *is not a root* of equation (A), but of its factor $\sqrt{x+1}+\sqrt{x}=\dfrac{-3}{\sqrt{1+x}}$, that *inevitably* results by the conditioned involution. Hence $x=\frac{4}{5}$ *only*.

SECOND (direct) "way." Transpose and square.

$\therefore x=x+1-6+\dfrac{9}{1+x}$. Thence $x=\frac{4}{5}$, [the "*same root as in the first way.*"]

PROOF. Apply *this* $x=\frac{4}{5}$, in equation (A).

$\therefore \sqrt{\frac{9}{5}}+\sqrt{\frac{4}{5}}=\dfrac{3\sqrt{5}}{\sqrt{9}}$. $\therefore$ as $\dfrac{3+2}{\sqrt{5}}=\dfrac{5}{\sqrt{5}}=\sqrt{5}$, so $\sqrt{5}=\dfrac{3\sqrt{5}}{\sqrt{9}}=\frac{3}{3}\sqrt{5}=1\sqrt{5}$,

i. e. $\sqrt{5}=\sqrt{5}$, *satisfies* equation (A). $\therefore x=\frac{4}{5}$ is the *one* root of (A). Q. E. D.

II. **Solution by J. SCHEFFER, A. M., Hagerstown, Md.**

Squaring the equation as it stands, we get $2x+1+2\sqrt{x(x+1)}=\frac{9}{x+1}$.

Clearing of fractions and leaving the radical by itself in the first member, we get $2(x+1)\sqrt{x(x+1)}=8-3x-2x^2$. Squaring, arranging, and cancelling, we get the quadratic $35x^2+52x=64$, the two roots of which are $x=\frac{4}{5}$ and $-\frac{16}{7}$, the the former of which satisfies the equation $\sqrt{x+1}+\sqrt{x}=\frac{3}{\sqrt{x+1}}$, and the latter the equation $\sqrt{x+1}-\sqrt{x}=\frac{3}{\sqrt{x+1}}$.

Clearing the original equation of its denominator $\sqrt{x+1}$, we have $x+1+\sqrt{x(x+1)}=3$, or $\sqrt{x(x+1)}=2-x$. Squaring, we have $5x=4$. $\therefore x=\frac{4}{5}$.

III. **Solution by F. M. McGAW, A. M., Professor of Mathematics in Bordentown Military Institute, Bordentown, N. J.; CHAS. C. CROSS, Laytonsville, Md.; G. B. M. ZERR, A. M., Ph. D., The Russell College, Lebanon, Va.; and J. P. BURDETTE, Class of '97, Dickinson College, Carlisle, Pa.**

(1), $\sqrt{x+1}+\sqrt{x}=\frac{3}{\sqrt{1+x}}$. $2x+1+2\sqrt{x+x^2}=\frac{9}{1+x}$.

$\therefore 8-2x^2-3x=2(x+1)\sqrt{x+x^2}$. $\therefore 35x^2+52x=64$. $\therefore x=\frac{4}{5}$, or $-2\frac{2}{7}$.

(2). Regarding $\sqrt{x+1}$ as affected by the $\pm$ sign

$\sqrt{x}=\frac{2-x}{\sqrt{1+x}}$ or $\frac{4+x}{\sqrt{1+x}}$.

$\therefore x=(4-4x+x^2)/(1+x)$, or $(16+8x+x^2)/(1+x)$.

$\therefore x=\frac{4}{5}$, or $x=-2\frac{2}{7}$.

Also solved by *A. H. BELL.*

75. **Proposed by the late B. F. BURLESON, Oneida Castle, N. Y.**

Mr. B's farm is in shape a quadrilateral, both inscriptible and circumscriptible, and contains an area of $k=10752$ square rods. The square described on the radius of its inscribed circle contains $r^2=2304$ square rods; while the square described on the radius of its circumscribed circle contains an area of $R^2=7345$ square rods. Required the lengths of the sides of his farm.

I. **Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.**

Let a, b, c, d be the sides required. By the conditions of the problem, $a+c=b+d$; $abcd=k^2=115605504$ ....(1).

$\frac{1}{2}r(a+b+c+d)=k$, or $a+b+c+d=2k/r=448$.

$\therefore a+c=b+d=k/r=224$ ....(2).

$$R=\tfrac{1}{4}\sqrt{\frac{(ab+cd)(ac+bd)(bc+ad)}{abcd}}.$$